



UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical &
Computer Engineering

ECE 204 *Numerical methods*

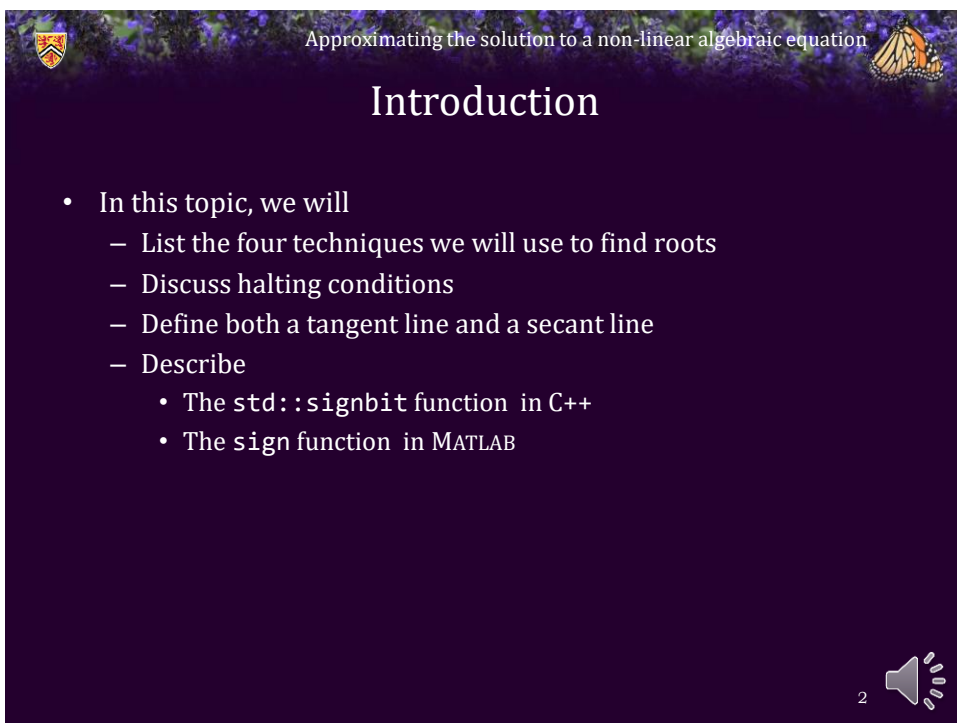
Approximating the solution to a non-linear algebraic equation

Douglas Wilhelm Harder, LEL, M.Math.
dwharder@waterloo.ca
dwharder@gmail.com

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Approximating the solution to a non-linear algebraic equation

Introduction

- In this topic, we will
 - List the four techniques we will use to find roots
 - Discuss halting conditions
 - Define both a tangent line and a secant line
 - Describe
 - The `std::signbit` function in C++
 - The `sign` function in MATLAB



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Approximating non-linear algebraic equations

- We continue by finding approximations to solutions to a non-linear equation
 - We convert any non-linear equation to a root-finding problem

$$f(x) = 0$$
- We will use seven approaches:

– Newton's method	Taylor series
– Bisection method	Bracketing
– Bracketed secant method	Bracketing and linear interpolation
– Secant method	Linear interpolation
– Muller's method	Quadratic interpolation
– Inverse quadratic interpolation	Quadratic interpolation
– Brent-Dekker method	2 nd , 3 rd and 6 th methods

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Approximating non-linear algebraic equations

- All these techniques will use iteration
 - What are the halting conditions?
 - We want to make sure we are likely close to a root

$$|x_{n+1} - x_n| < \epsilon_{\text{step}}$$
 - We want to make sure it is a root and not a discontinuity

$$|f(x_{n+1})| < \epsilon_{\text{abs}}$$

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Solutions to equations

- Reviewing definitions:
 - A tangent line is a line that touches a curve at one point
 - A secant line is one that intersects a curve at two points

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Solutions to equations

- A first-order Taylor series approximation defines a tangent line to a point on a curve
 - This assumes the function is differentiable at the point
 - It may intersect the curve elsewhere, but at the point it is tangent
- A linear interpolating polynomial defines a secant line

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Equal or opposite signs?

- In C++, there is the `std::signbit(double x)` function
 - It is in the `cmath` library
 - It returns the sign bit as a Boolean value
 - If the sign bit is 1, the number is negative and `true` is returned
 - Otherwise, the sign bit is 0, the number is positive, and `false` is returned
- We test if two variables `x` and `y` have the same sign with


```
if ( std::signbit( x ) == std::signbit( y ) ) {
    // do something
}
```



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Equal or opposite signs?

- In MATLAB, there is the `sign(x)` function
 - It returns:
 - 0 if the argument is zero
 - 1 if the argument is positive
 - 1 if the argument is negative
- We test if two non-zero variables `x` and `y` have the same sign with


```
if sign( x ) == sign( y )
    % do something
end
```




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Summary

- Following this topic, you now
 - Understand we are looking at four root-finding techniques
 - Each uses different tools to find the root
 - Each uses iteration
 - Understand that we have two halting conditions, both of which must be satisfied
 - Have reviewed the definitions of a tangent line and secant line
 - Are aware of the `std::signbit(...)` function and the `sign(...)`


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References

[1] https://en.wikipedia.org/wiki/Nonlinear_system

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Acknowledgments

None so far.

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Colophon


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